17. E. F. Afanas'ev and V. N. Nikolaevskii, "Construction of asymmetrical hydrodynamics on suspensions with rotating solid particles," in: Problems of Hydrodynamics and Mechanics of a Continuous Medium [in Russian], Nauka, Moscow (1969), pp. 17-24.
18. V. N. Nikolaevskii, "Stress tensor and averaging in mechanics of continuous media," Prikl. Mat. Mekh., 39, 374-379 (1975).
19. H. Immich, "Zweiphasen-kontinuumstheorie einer Suspension starren, kugelförmigen Teilchen unter Berücksichtigung der Teilchenrotation. Teils I und II," Z. Angew. Math. Mech., 60, No. 2, 99-107 (1980); 60, No. 3, 153-160 (1980).
20. V. N. Nikolaevskii, "Short note on the space averaging in continuum mechanics," Int. J. Multiphase Flow, 6, No. 2, 375-378 (1980).
21. R. I. Nigmatulin, Comments to "Short note on the space averaging in continuum mechanics" by V. N. Nikolaevskii, Int. J. Multiphase Flow, 6, No. 2, 379-382 (1980).

SETTLING OF A BIDISPERSE SUSPENSION

B. S. Endler

UDC 66.063.94

The influence of the fractional composition on the settling velocity of a bidisperse suspension is investigated theoretically. The average particle radius of a settling bidisperse suspension is calculated.

Monodisperse suspensions are rarely encountered in practical engineering, and a disperse phase usually consists of a mixture of particles of different sizes. Despite this, considerably fewer reports have been devoted to the investigation of the motion of polydisperse than of monodisperse suspensions.

The force of interaction between the liquid and disperse phases of a polydisperse suspension of moderate concentration was determined in [1] using rigorous statistical methods. Without additional considerations, however, one cannot determine from [1] the velocities of motion of the separate fractions needed to study the settling of a polydisperse suspension.

The settling of multifraction suspensions of fine particles of equal density was investigated theoretically in [2-6] on the basis of various assumptions about the form of the dependence of these velocities on the fractional composition and the total volumetric concentration of the disperse phase. Here, by analogy with the monodisperse case, the dependence of the settling velocities of the individual fractions on the fractional composition and the total concentration was assigned in [2, 3] in power-law form, where the porosity of the suspension served as the base while the exponent depended on the composition. A modified cell model was used for these purposes in [4], and data of [7] on the magnitude of the force of interaction between a filtering stream and a stationary, polydisperse granular bed were used in [5, 6]. The settling of bidisperse suspensions of particles of equal density at low Reynolds numbers was investigated experimentally in [2-5, 8-11].

Let us consider the uniform gravitational settling of a bidisperse suspension of moderate concentration. The continuous phase consists of an incompressible Newtonian liquid with a viscosity μ_0 and density d_0, while the disperse phase consists of a mixture of two fractions of spherical particles with radii a' and a'' and a density d_1. The volumetric concentrations of the particles a' and a'' and of the entire disperse phase are ρ', ρ'', and $\rho = \rho' + \rho''$, respectively. The Reynolds numbers characterizing both the flow over individual particles and the motion of the phases on the average are small compared with unity. For determinacy, let $a'' > a'$ and let the distribution of the concentrations ρ' and ρ'' be uniform.

Using [1], we represent the force of interaction of each fraction with the continuous phase, due to the action of viscous-friction forces, as the sum of two terms: the first, allowing for the constrained nature of the settling, coincides with that of [1]; the second characterizes the interaction between the fractions, due to the difference between the settling velocities of the fractions. Then the equations of conservation of mass and momentum describing the settling of the fractions have the form

---

All-Union Scientific-Research Institute of Petrochemical Processes, Leningrad. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 44, No. 4, pp. 601-607, April, 1983. Original article submitted November 23, 1981.

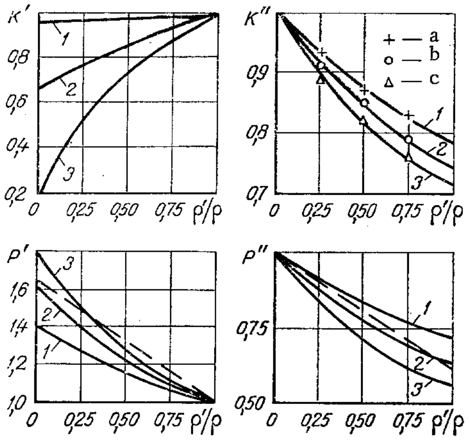

Fig. 1. Dependence of the coefficients K', K", P', and P" on the fractional composition of the disperse phase. Solid curves) 1) ρ = 0.1; 2) 0.2; 3) 0.3; dashed curves) data of [11]; points) experiment of [10]: a) for ρ = 0.1; b) 0.2; c) 0.3.

$$\begin{gathered}
\nabla \mathbf{v}' = 0; \quad \nabla \mathbf{v}'' = 0; \\
\frac{9}{2}\,\frac{\mu_0}{(a')^2}\,\rho' F(\mathbf{v}_0 - \mathbf{v}') + \frac{9}{2}\,\frac{\mu_0}{(a')^2}\,\rho'\rho''\lambda'(\mathbf{v}'' - \mathbf{v}') = -\rho'(d - d_0)\,g; \\
\frac{9}{2}\,\frac{\mu_0}{(a'')^2}\,\rho'' F(\mathbf{v}_0 - \mathbf{v}'') + \frac{9}{2}\,\frac{\mu_0}{(a'')^2}\,\rho'\rho''\lambda''(\mathbf{v}' - \mathbf{v}'') = -\rho''(d - d_0)\,g; \\
F = (1 - 2.5\rho)^{-1}; \quad d = \varepsilon d_0 + \rho d_1; \quad \varepsilon = 1 - \rho.
\end{gathered} \tag{1}$$

The structure of the expression for the second terms on the left sides of the equations of conservation of momentum (1) is determined from the following considerations. In the cosettling of two particle fractions of different sizes and the same density one of the fractions passes through the layer of particles of the other fraction. In the case when the concentration of the disperse phase is moderate, the force of interaction between a certain particle of one size and the fraction of particles of the other size can be assumed to be proportional to the volumetric concentration of particles of the other size. Then the force of interaction between fractions must be proportional to the product of the volumetric concentrations of the fractions, $\rho'\rho''$, and the difference between their settling velocities. Because the forces of interaction between the fractions are equal, the coefficients λ' and λ'' must satisfy the relation

$$\lambda'/(a')^2 = \lambda''(a'')^2. \tag{2}$$

Let us consider a settling process in which the velocity of motion of the suspension as a whole, $\mathbf{v} = \varepsilon \mathbf{v}_0 + \rho'\mathbf{v}' + \rho''\mathbf{v}''$, equals zero, the motion of the phases takes place in the vertical direction, and all the variables depend only on the vertical coordinate. With $\mathbf{v} = 0$ and a uniform initial particle distribution over the height of the suspension column, four characteristic zones are formed after a certain time following the start of settling: 0) a zone of pure water; 1) a zone of settling of particles of radius a'; 2) a zone of cosettling of particles of radii a' and a''; 3) a sediment zone. The volumetric concentrations ρ' and ρ'' in the second zone do not vary with time and hence equal the initial concentrations.

Using (1) and (2) we obtain the expressions for the settling velocities of the fractions of fine ($\mathbf{v}_2'$) and coarse ($\mathbf{v}_2''$) particles in the second zone:

$$\begin{gathered}
\mathbf{v}_2' = K'\mathbf{u}'; \quad K' = \frac{\varepsilon + \rho''(1 - s^2) + \rho s^2 \varepsilon \lambda' F^{-1}}{\varepsilon[1 + (\rho'' + \rho' s^2)\lambda' F^{-1}]}; \\
\mathbf{v}_2'' = K''\mathbf{u}''; \quad K'' = \frac{\varepsilon + \rho'(1 - s^{-2}) + \varepsilon\rho\lambda' F^{-1}}{\varepsilon[1 + (\rho'' + \rho' s^2)\lambda' F^{-1}]}.
\end{gathered} \tag{3}$$

Here $s = a''/a'$ and $\mathbf{u}'$ and $\mathbf{u}''$ are the settling velocities of disperse phases of monodisperse suspensions of particles a' and a'', respectively, at the concentration $\rho = \rho' + \rho''$ and under the condition that the suspension as a whole is stationary [1]:

$$\mathbf{u}' = (1 - \rho)^2(1 - 2{,}5\rho)\,\mathbf{u}_1; \quad \mathbf{u}'' = (1 - \rho)^2(1 - 2.5\rho)\,\mathbf{u}_2,$$

where $\mathbf{u}_1$ and $\mathbf{u}_2$ are the settling velocities of single particles a' and a'', respectively, in an unbounded stationary liquid. The coefficients K' and K" show the difference between the settling velocities in the bi- and monodisperse cases. For s = 1 ($a' = a''$) we have K' = K" = 1.

By analogy with (3), from (1) and (2) we obtain expressions for the velocities of motion of the fractions relative to the liquid in the second zone:

$$\mathbf{v}_2' - \mathbf{v}_0 = P'(\mathbf{u}' - \mathbf{u}_0); \quad P' = \frac{F + \lambda'\rho s^2}{F + \lambda'(\rho'' + s^2\rho')};$$
$$\mathbf{v}_2'' - \mathbf{v}_0 = P''(\mathbf{u}'' - \mathbf{u}_0); \quad P'' = \frac{F + \lambda'\rho}{F + \lambda'(\rho'' + s^2\rho')}. \tag{4}$$

The coefficients P' and P" characterize the difference between the velocities of motion of the fractions relative to the liquid in bi- and monodisperse suspensions.

The settling velocity of particles of radius a' in the first zone is (see (3))

$$\mathbf{v}_1' = (1 - \rho_1')^2 (1 - 2{,}5\,\rho_1')\,\mathbf{u}_1. \tag{5}$$

Equations (3)-(5), together with the relations

$$-\mathbf{v}_1' = \varepsilon_1(\mathbf{v}_{01} - \mathbf{v}_1'); \quad \varepsilon_1(\mathbf{v}_{01} - \mathbf{v}_2'') = \varepsilon(\mathbf{v}_{02} - \mathbf{v}_2'');$$
$$\rho_1'(\mathbf{v}_1' - \mathbf{v}_2'') = \rho'(\mathbf{v}_2' - \mathbf{v}_2''), \tag{6}$$

which follow from the conditions of conservation of mass in the transition through the boundaries of the zones, determine the velocities v_1', v_2', v_2'', v_{01}, and v_{02} and the concentration ρ_1' for given initial concentrations ρ' and ρ'' and a known value of the coefficient λ'.

The first equation of (6) determines the flux of the continuous phase from the first to the zeroth zone while the second and third equations determine the fluxes of the continuous phases and the fraction of fine particles, respectively, from the second to the first zone. The third equation of (6) shows that the concentration of fine particles in the first zone will be higher than in the initial mixture ($\rho_1' > \rho'$) if the fine particles settle faster in the first zone than in the second ($v_1' > v_2'$).

To close equations (3)-(6) we must know λ'.

Let $\lambda' = \lambda'' = 0$. Then from (3)-(6) it follows that in a bidisperse suspension fine particles settle slower ($K' < 1$) and large particles settle faster ($K'' > 1$) than in the corresponding monodisperse suspensions with $\rho = \text{const}$. And when the condition $(1 - \rho)/(\rho - \rho') < s^2 - 1$ is satisfied the coefficient K' is negative, which corresponds to upward motion of fine particles in the second zone under the action of the ascending liquid flow. The velocities of the coarse and fine particles relative to the liquid do not depend on the fractional composition of the disperse phase and equal the analogous velocities in monodisperse suspensions. The settling velocity of fine particles in the second zone is lower than in the first zone, and hence the concentration of these particles is higher in the first zone than in the initial mixture.

Let $\lambda' \gg 1$ ($\lambda'' \gg 1$). Then, with allowance for the relation $u'' = s^2u'$, it follows from (3) and (4) that: particles of different sizes settle at the same velocities, i.e., in a regime of consolidated settling; the first zone does not exist; in a bidisperse suspension fine particles settle faster both relative to the walls and the liquid ($P' = K' > 1$), while coarse particles settle slower ($P'' = K'' < 1$), than in the corresponding monodisperse suspensions with $\rho = \text{const}$. The consolidated regime of settling of a bidisperse suspension was observed in the experiments of [9] with highly concentrated suspensions ($\rho > 0.3$). The reason for the strong interaction between the fractions may be, for example, contact interactions between particles. However, to describe effects of this kind one must remove the "paradox of zero relative velocity" [12], according to which, under the assumption that the Reynolds numbers are small, two particles under the action of gravity can come together up to contact only in an infinite time.

To determine the true values of the coefficients λ' and λ" we used the experiments of [10], in which the influence of the fractional composition of the disperse phase of a bidisperse suspension on the settling of the fraction of coarse particles was studied for constant values of a' and a''.

A comparison of the results of the calculation of K" with the experimental data of [10] showed (Fig. 1) that in the range of variation of the concentration of the disperse phase of 0-0.3 and for a fixed value of the parameter s the coefficient λ' does not depend on the fractional composition, being a function of the concentration of the disperse phase as a whole, and its behavior is described by the formula

$$\lambda' = 1.289 + \exp(17.84\,\rho - 4.778). \tag{7}$$

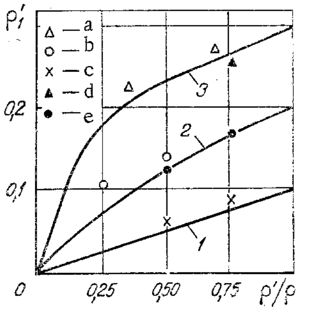

Fig. 2

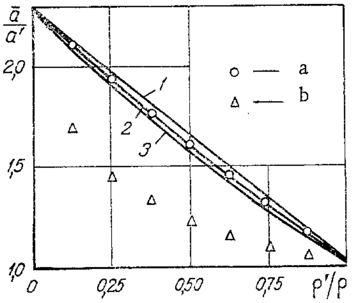

Fig. 3

Fig. 2. Dependence of concentration ρ_1' of fine particles in the first zone on the fractional composition of the disperse phase. Curves) calculation: 1) $\rho = 0.1$; 2) 0.2; 3) 0.3; points) experiment [5]: c) for $\rho = 0.1$; b, c) 0.2; a, e) for $\rho = 0.3$ (a, b, c: polystyrene spheres; d, e: quartz sand).

Fig. 3. Dependence of $\bar{a}/a'$ on the fractional composition ρ'/ρ of the disperse phase: curves) Eq. (8): 1) $\rho = 0.1$; 2) 0.2; 3) 0.3; a) approximating equation (9); b) data of [15].

We obtain the value of the coefficient λ'' from (2) with $s = 2.285$.

Using (7), we calculated the coefficients K', P', and P'' (see Fig. 1) and the concentration ρ_1' of fine particles in the first zone (Fig. 2). A graph of the dependence from [11] is also presented in Fig. 1.

An analysis of the data presented in Fig. 1 shows that the coarse particles in a bidisperse suspension move slower relative to the walls of the vessel containing the suspension and relative to the liquid than in a monodisperse suspension of large particles with the same concentration of the disperse phase as a whole ($K'' < 1$, $P'' < 1$), and $P'' < K''$ for fixed values of ρ'/ρ and ρ. Conversely, fine particles in a bidisperse suspension move faster relative to the liquid ($P' > 1$) and slower relative to the wall ($K' < 1$) than in a monodisperse suspension of fine particles with a fixed value of ρ.

The results of the calculations, which are well confirmed experimentally (see Fig. 2), indicate an increase in the concentration of fine particles in the first zone compared with the concentration of these particles in the initial suspension. This means that the settling velocity of fine particles is lower in the first zone than in the second.

The actual settling regime is intermediate to the limiting settling regimes analyzed above with negligibly small ($\lambda' = \lambda'' = 0$) and strong ($\lambda' \gg 1$, $\lambda'' \gg 1$) interaction between phases.

Average Particle Radius of a Settling Bidisperse Suspension. To make engineering calculations of the flow of a polydisperse suspension, analyze test data on its settling, etc., one must assign the value of the average particle radius. There is no unique means of choosing such a radius in the literature. For example, in [13] the average radius is determined from the arithmetic-mean velocity of settling of 150 identical particles, in [14] the volumetric-mean radius was chosen as the average, and in [15] the average radius was represented in the form of a combination of the first three moments of the particle size on the basis of an analysis of the settling of a polydisperse suspension of particles not interacting with each other.

Here the average radius is determined from the obvious condition that the settling velocities of a polydisperse suspension and a monodisperse suspension of particles of the average radius coincide.

Using (3) we represent the volumetric flux of the disperse phase of a bidisperse suspension, $\rho v_1 = \rho' v' + \rho'' v''$, in the form

$$\rho v_1 = \rho u' A, \quad A = \frac{\rho'}{\rho} K' + s^2 \frac{\rho''}{\rho} K''. \tag{8}$$

We introduce the radius $\bar{a}$ in accordance with the relation $\bar{a} = a' \sqrt{A}$. Then it follows from (8) that the volumetric settling velocities of the disperse phases of a bidisperse suspension and a monodisperse suspension of particles of radius $\bar{a}$ are equal, and hence the radius $\bar{a}$ is the average radius. A graph of the dependence of $\bar{a}/a'$ on the fractional composition ρ'/ρ and the volumetric concentration of the disperse phase of a bidisperse suspension is presented in Fig. 3.

In engineering calculations the average radius is usually expressed through a combination of i-th moments of particle size.

$$m_i = \int a^i \varphi(a)\, da.$$

A comparison of the two means of representing the average radius $\bar{a}$ defined by (8) is approximated most satisfactorily by the quantity

$$\bar{a} = \frac{m_4}{m_3} = a'\left(\frac{\rho'}{\rho} + s\frac{\rho''}{\rho}\right). \tag{9}$$

A graph of the function $\bar{a}/a'$ given by (9) is shown in Fig. 3. The results of a calculation from the equation proposed in [15] are also illustrated there. It is seen from Fig. 3 that the function (9) yields better results than the expression from [15].

From an analysis of the results obtained it follows that the fractional composition of the disperse phase affects the settling velocity of the disperse phase, and this influence increases with an increase in the total concentration. In periodic settling the coarse and fine particles in a bidisperse suspension move slower relative to the vessel walls than in periodic settling of monodisperse suspensions of the respective coarse and fine particles with equal values of the concentration of the disperse phase as a whole. However, fine particles move faster relative to the liquid and coarse ones move slower in the case of a bidisperse suspension than in the respective monodisperse suspensions. In periodic settling of a bidisperse suspension the concentration of fine particles in the upper zone is higher than in the initial suspension. The volumetric settling velocities of the disperse phases of bi- and monodisperse suspensions coincide if the particles of the monodisperse suspension have a radius equal to the ratio of the fourth and third moments of the particle size of the disperse phase of the bidisperse suspension.

NOTATION

μ_0, d_0, liquid viscosity and density; d_1, density of particle material; a', a'', particle radii; v', v", velocities of fractions of particles a' and a''; v_0 and v_1, velocities of continuous and disperse phases; v_{01}, v_{02}, velocities of continuous phase in the first and second zones, respectively; v_1', ρ_1', ε_1, velocity and volumetric concentration of fraction of particles a' and porosity in the first zone; v_2', v_2'', settling velocities of fractions of particles a' and a'' in the second zone; ρ', ρ'', volumetric concentrations of particles a' and a'' in the initial mixture and in the second zone; $\rho = \rho' + \rho''$; ε, volumetric concentration of the disperse phase as a whole and porosity of the suspension in the initial mixture and in the second zone; g, free-fall acceleration; F, λ', λ'', proportionality factors in (1); K', K", P', P", coefficients in (3) and (4); u_0, velocity of continuous phase of the monodisperse suspension; u' and u", settling velocities of disperse phases of monodisperse suspensions of particles a' and a''; u_1 and u_2, settling velocities of single particles a' and a''; $\bar{a}$, average radius; A, coefficient in (8); m_i, i-th moment of particle size; $\varphi(a)$, particle size distribution function; $s = a''/a'$, parameter.

LITERATURE CITED

1. Yu. A. Buevich and I. N. Shchelchkova, "Rheological properties of uniform finely disperse suspensions: steady flows," Inzh.-Fiz. Zh., 33, No. 5, 872-879 (1977).
2. S. Mirza and J. F. Richardson, "Sedimentation of suspensions of particles of two or more sizes," Chem. Eng. Sci., 34, No. 4, 447-454 (1979).
3. M. J. Lockett and K. S. Basson, "Sedimentation of binary particles mixtures," Powder Technol., 24, No. 1, 1-7 (1979).
4. T. N. Smith, "The differential sedimentation of particles of two different species," Trans. Inst. Chem. Eng., 43, No. 3, 69-73 (1965).
5. M. B. Avrutov and B. S. Endler, "Particle distribution over the height of a vessel in the periodic settling of polydisperse suspensions," Teor. Osn. Khim. Tekhnol., 9, No. 6, 911-943 (1975).
6. V. M. Livintsev and A. E. Mozol'kov, "On the behavior of a polydisperse cloud of particles in a gas stream at low Reynolds numbers," Inzh.-Fiz. Zh., 30, No. 2, 235-239 (1976).
7. C. K. W. Tam, "The drag on a cloud of spherical particles in low Reynolds number flow," J. Fluid Mech., 38, No. 3, 537-546 (1969).
8. M. J. Lockett and H. M. Al-Habbooby, "Differential settling by size of two particle species in a liquid," Trans. Inst. Chem. Eng., 51, 281-292 (1973).
9. R. Davis, "The experimental study of the differential settling of particles in suspensions at high concentrations," Powder Technol., 2, 43-51 (1968/69).

10. M. B. Avrutov and B. S. Endler, "Periodic settling of a polydisperse suspension," Teor. Osn. Khim. Technol., 12, No. 4, 627-628 (1978).
11. B. V. Kizeval'tr, Theoretical Principles of Gravitational Enrichment Processes [in Russian], Nedra, Moscow (1979).
12. V. M. Voloshchuk and Yu. S. Sedunov, Coagulation Processes in Disperse Systems [in Russian], Gidrometeoizdat, Leningrad (1975).
13. H. H. Steinour, "Nonflocculated suspensions of uniform spheres," Ind. Eng. Chem., 36, No. 7, 618-624 (1944).
14. T. S. Mertes and H. B. Rhodes, "Liquid-particle behaviour," Chem. Eng. Prog., No. 9, 429-432, 517-522 (1955).
15. K. J. Scott and W. G. B. Mandersloot, "The mean particle size in hindered settling of multisized particles," Powder Technol., 24, No. 1, 99-101 (1979).

SIMULATION OF VERTICAL TRANSPORT OF HEAT AND SOLID PARTICLES IN FLUIDIZED-BED APPARATUS

Yu. S. Teplitskii and I. I. Yanovich

UDC 66.096.5

An equation for simulating nonstationary vertical transport of heat and solid particles in nonhomogeneous fluidized beds is proposed.

The trends in the transport of solid particles and the related heat transport* throughout the fluidized bed space strongly affect the operating efficiency of apparatus based on fluidization techniques. Therefore, the mixing of particles and internal heat transfer in such a system invariably engage the interest of researchers [1-4].

Until recently, the most commonly accepted vertical mixing scheme was based on the classical diffusion model, which describes the process by means of a single parameter — the coefficients of vertical diffusion (dispersion) of particles [1]. However, this model cannot describe the experimentally observed nonstationary mixing curves [5].

It was proposed in [6], probably for the first time, to describe the process by a hyperbolic diffusion equation that would take into account the finiteness of the particle velocities. A system of two hyperbolic first-order equations was used earlier in [7] for describing the vertical mixing of the solid phase. This system was based on the assumption that the transport of particles throughout the bed was purely convective (circulatory) in character: upward in the bubble trails and downward in the rest of the emulsion phase. In this, the particle velocities in both phases were, naturally, limited. The necessity and importance of taking into account the finiteness of the velocity of particles was shown in [5, 8] by direct comparison between the experimental mixing curves and those calculated by means of hyperbolic equations [6, 7]. Using the results of an analysis of the fluidization process based on methods of the thermodynamics of irreversible processes, Liu and Gidaspow have derived [3] a hyperbolic equation of diffusion to describe vertical solid phase transport. It has been suggested in [4] to use three first-order hyperbolic equations to describe vertical mixing of particles in a bed slowed down by a bunch of pipes. An additional equation (in comparison with the system given in [7]) describes the downward core motion of particles at the wall. Analysis shows that none of the above models comprises all the basic characteristics of the mixing process (see below).

---

* It is admissible to assume that the heat transfer is due entirely to the motion of particles in nonhomogeneous fluidized beds because of the large difference between the volumetric specific heat values of the gas and the particles. The transport of heat and the transport of disperse material and therefore characterized by the same trends, so that, for brevity, we shall subsequently make no special distinction between these processes and use only the term "mixing" (diffusion of particles) or "thermal conductivity" of the bed.

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 44, No. 4, pp. 608-615, April, 1983. Original article submitted December 7, 1981.